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Piezomagnetic field associated with a vertical rectangular strike-slip or tensile fault revisited: Elimination of apparent singularities at fault edges

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Abstract

Sasai in 1991 presented analytic formulas for the piezomagnetic field associated with a vertical rectangular fault with strike-slip or tensile fault motion. There appear apparently divergent terms when we approach fault edges on the ground surface. They are those containing $1/r^2$, $1/r^4$ and $1/r^6$, where r implies the horizontal distance between the observation point and fault edges. However, we find that these terms are convergent to finite values by combining appropriate two terms in the Chinnery sum. The Tailor series expansion can be defined for field values around the fault edges.

1 Introduction

Sasai (1991) formulated the Green's function method in tectonomagnetic modeling. It was applied to derive analytic solutions for the piezomagnetic field associated with a vertical rectangular fault with strike-slip or tensile fault motion. The solutions are given for their magnetic potentials in the Appendix E in his paper. They are not given here for the sake of brevity. In his solutions of the magnetic potential are included terms containing $1/r^2$ and $1/r^4$, where r implies the horizontal distance between the observation point and the two fault edges. Differentiating potentials, we obtain the piezomagnetic field containing $1/r^2$, $1/r^4$ and $1/r^6$. We call them divergent terms. However, we will find that such divergent characteristics are simply apparent and that they remain finite at both the fault edges.

2 Chinnery sum

The piezomagnetic potentials for the strike-slip or tensile faulting are given in terms of Chinnery sum. This notaion was first introduced by Chinnery (1961) to represent the displacement field associated with a vertical rectangular strike-slip fault. Each term consists of the combination of four terms:

$$\{f(x_1, x_3)\} \|_a^b = f(L, b) - f(L, a) - f(-L, b) + f(-L, a) \tag{1}$$

where $x_1 = \pm L$ indicates the horizontal position of two fault edges and $x_3 = a$ and b implies the depth of the fault top d and the bottom D. In Sasai's solution, D is sometimes replaced with H (Curie depth), depending on the spatial configuration of the fault bottom (D) and the Curie depth (H).

We use the following relationship among the observation point (x, y, z) and the fault position (x_1, x_3):

$$t = x - x_1,\ r^2 = t^2 + y^2, \tag{2}$$

$$p_1 = x_3 - z,\ p_2 = 2H - x_3 - z,\ p_3 = 2H + x_3 - z, \tag{3}$$

$$S_1 = \sqrt{r^2 + p_1^2},\ S_2 = \sqrt{r^2 + p_2^2},\ S_3 = \sqrt{r^2 + p_3^2} \tag{4}$$

r in eq. (2) tends to be zero when we approach both the fault edges. The potential terms to produce divergent ones are

$$\{\tan^{-1}(\frac{tp}{yS})\} \|_a^b, \quad \{\frac{1}{r^2}\frac{p}{S+p}\} \|_a^b, \quad \{\frac{1}{r^2}\frac{p^2}{S(S+p)}\} \|_a^b, \quad \{\frac{1}{r^2}\frac{p}{S}\} \|_a^b,$$

and

$$\{\frac{1}{r^4}(\frac{3p}{S} - \frac{p^3}{S^3})\} \|_a^b, \quad \{\frac{1}{r^4}(\frac{3y^2 - r^2}{r^2}\frac{p}{S} - \frac{y^2}{r^2}\frac{p^3}{S^3})\} \|_a^b$$

Differentiating these potentials, we finally find the following 7 divergent functions in the piezomagnetic field.

$$F_1 = \frac{1}{r^2}\{\frac{p}{S}\} \|_a^b \tag{5}$$

$$F_2 = \frac{1}{r^2}\{\frac{p}{S+p}\} \|_a^b \tag{6}$$

$$F_3 = \frac{1}{r^2}\{\frac{p^2}{S(S+p)}\} \|_a^b \tag{7}$$

$$F_4 = \frac{1}{r^4}\{\frac{3p}{S+p} - \frac{p^3}{S^3}\} \|_a^b \tag{8}$$

$$F_5 = \frac{1}{r^4}\{\frac{3p}{S+p} - \frac{p^2}{S(S+p)}\} \|_a^b \tag{9}$$

$$F_6 = \frac{1}{r^4}\{\frac{4p^2}{S(S+p)} - \frac{p^3}{S^3}\} \|_a^b \tag{10}$$

$$F_7 = \frac{1}{r^6}\{\frac{15p}{S} - \frac{10p^3}{S^3} + \frac{3p^5}{S^5}\} \|_a^b \tag{11}$$

The suffix shown in eq. (3) is omitted in the above.

3 Tailor series expansion of divergent terms

Now let us consider the combination of the 1st and 2nd term of the Chinnery sum (1). When the observation point is located near around $+L$, the sum of the 1st and 2nd term has the denominator r^n ($n = 2, 4, 6$) and its numerators are given as follows:

$$f_1(r) = \frac{p_b}{S_b} - \frac{p_a}{S_a} \tag{12}$$

$$f_2(r) = \frac{p_b}{S_b + p_b} - \frac{p_a}{S_a + p_a} \tag{13}$$

$$f_3(r) = \frac{p_b^2}{S_b(S_b + p_b)} - \frac{p_a^2}{S_a(S_a + p_a)} \tag{14}$$

$$f_4(r) = (\frac{3p_b}{S_b}) - \frac{p_b^3}{S_b^3}) - (\frac{3p_a}{S_a} - \frac{p_a^3}{S_a^3}) \tag{15}$$

$$f_5(r) = (\frac{3p_b}{S_b + p_b} - \frac{p_b^2}{S_b(S_b + p_b)}) - (\frac{3p_a}{S_a + p_a} - \frac{p_a^2}{S_a(S_a + p_a)}) \tag{16}$$

$$f_6(r) = (\frac{4p_b^2}{S_b(S_b + p_b)} - \frac{p_b^3}{S_b^3}) - (\frac{4p_a^2}{S_a(S_a + p_a)} - \frac{p_a^3}{S_a^3}) \tag{17}$$

$$f_7(r) = (\frac{15p_b}{S_b} - \frac{10p_b^3}{S_b^3} + \frac{3p_b^5}{S_b^5}) - (\frac{15p_a}{S_a} - \frac{10p_a^3}{S_a^3} + \frac{3p_a^5}{S_a^5}) \tag{18}$$

Obviously, the numerators (12) to (18) become zero for $r = 0$. Then the functions F_1 to F_7 have the indeterminate form of convergence of the type 0/0, i.e. zero devided by zero. We have a well-known method to obtain such kind of indeterminate limit (Moriguchi et al., 1956). We may differentiate the denominator and the numerator with repect to r. We can find the finite limit of the divergent functions for $r \to 0$. Similarly, we can find the finite values of the piezomagnetic field at another fault edge, i.e. $x_1 = -L$, by combining the third and fourth term of Chinnery sum.

As for the field values near around the fault edges $x_1 = \pm L$, we can define the Tailor series expansion of the divergent terms with respect to r. We simply give the formulas in the following:

$$F_1 = \frac{1}{r^2}\frac{p}{S} = -\frac{1}{2}A + \frac{3}{8}Br^2 - \frac{5}{16}Cr^4 + \dots \tag{19}$$

$$F_2 = \frac{1}{r^2}\frac{p}{S + p} = -\frac{1}{8}A + \frac{1}{16}Br^2 - \frac{5}{128}Cr^4 + \dots \tag{20}$$

$$F_3 = \frac{1}{r^4}\left(\frac{3p}{S} - \frac{p^3}{S^3}\right) = -\frac{3}{4}B + \frac{5}{4}Cr^2 - \frac{105}{64}Dr^4 + \dots \tag{21}$$

$$F_4 = \frac{1}{r^4}\left(\frac{3p}{S + p} - \frac{p^2}{S(S + p)}\right) = -\frac{1}{8}B + \frac{5}{32}Cr^2 - \frac{21}{128}Dr^4 + \dots \tag{22}$$

$$F_5 = \frac{1}{r^4}\left(\frac{4p^2}{S(S + p)} - \frac{p^3}{S^3}\right) = -\frac{5}{8}B + \frac{35}{32}Cr^2 - \frac{189}{128}Dr^4 + \dots \tag{23}$$

$$F_6 = \frac{1}{r^2}\frac{p^2}{S(S + p)} = -\frac{3}{8}A + \frac{5}{16}Br^2 - \frac{35}{128}Cr^4 + \dots \tag{24}$$

$$F_7 = \frac{1}{r^6}\left(\frac{15p}{S} - \frac{10p^3}{S^3} + \frac{3p^5}{S^5}\right) = -\frac{5}{2}C + \frac{105}{16}Dr^2 - \frac{189}{16}Er^4 + \dots \tag{25}$$

where

$$A = \frac{1}{p_n^2(a)} - \frac{1}{p_n^2(b)} \tag{26}$$

$$B = \frac{1}{p_n^4(a)} - \frac{1}{p_n^4(b)} \tag{27}$$

$$C = \frac{1}{p_n^6(a)} - \frac{1}{p_n^6(b)} \tag{28}$$

$$D = \frac{1}{p_n^8(a)} - \frac{1}{p_n^8(b)} \tag{29}$$

$$E = \frac{1}{p_n^{10}(a)} - \frac{1}{p_n^{10}(b)} \tag{30}$$

and $p_n(x_3) \quad (n = 1, 2, 3)$ corresponds to three different terms in eq. (3). We apply the above formulas for $r \leq L/10$.

4 Discussion

In Sasai's (1991) paper, the piezomagnetic fields over the ground surface are shown for the strike-slip and tensile faults. There appear no singularities around the fault edges. This is because the field values at the fault edges were already known thanks to the indeterminate procedure. However, a more close look at piezomagnetic fields at fault edges was required when we discuss the observations near the Landers, California, earthquake of Mw 7.3 (Johnston et al., 1994). I summarized the results after 20 years in this manuscript. Utsugi et al.'s (2000) formula for inclined rectangular faults is now widely used by many researchers, in which the fault inclination is limited up to 89°. Their results are compared with Sasai's (1991) figures. The rigorous solution for the fault inclination 90° is given here. This paper would be of some help to young researchers who are interested in tectonomagnetism.

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