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Estimation of mean one-dimensional model from heterogenous magnetotelluric impedances

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Abstract

Fast and stable convergence of forward calculation is essentially important for three-dimensional (3-D) magnetotelluric (MT) inversion problems to be practical. And this requires the condition number of the system matrix of the forward problem to be as small as possible. It implies that the system matrix will be well-conditioned, if we use a mean one-dimensional (1-D) model as a background. So the problem becomes how to estimate the mean 1-D model, especially in case of seafloor MT data which we are mostly working on. We propose an approach that allows a direct estimation of mean 1-D model from heterogeneous MT impedances. We assume a number of MT observations were made on the surface of the Earth consisting of a 3-D heterogeneous surface layer and 1-D structure below. We tested this new method by using synthetic models and compared the inversion result and its misfit. The result of synthetic tests indicates sufficiently high performance and reliability of this method.

Introduction

Avdeev (2006) showed that the appropriate preconditioned system matrix would better have small condition number, no larger than (Cl)^1/2, where Cl is a lateral contrast of conductivity. This is necessary to get stable and fast convergence in the inversion problem. It means the system matrix will be well-conditioned, if we use a mean one-dimensional (1-D) model as a background in the starting model.

Thus, the problem becomes how to estimate the mean 1-D model, especially in case of seafloor MT data we mostly concentrate on. One possible way of this sort of estimation was proposed in the work by Baba et al. (2010), in which the optimum 1-D model is estimated iteratively from spatially averaged rotationally invariant impedance. In this article we propose a new approach, which could directly estimate mean 1-D model from heterogeneous MT impedances below undulating seafloor.

Forward modeling

The electromagnetic field can be described by Maxwell equation (1). General 3-D conductivity distribution is separated into the mean 1-D structure and perturbation (lateral variation). Then the electromagnetic field can be divided into the primary field and the secondary (scattered) field which satisfy equations (2) and (3), respectively. In these equations, there are two electric current sources, $\mathbf{J}^p$ and $\mathbf{J}^q$. $\mathbf{J}^p$ is a primary source, which is due to the external (ionosphere/magnetosphere) current system in case of magnetotellurics. $\mathbf{J}^q$ is a secondary source given by equation (4)[2-3].

$$\begin{aligned} &\nabla \times \mathbf{E} - i\omega\mu_0 \mathbf{H} = 0 \\ &\nabla \times \mathbf{H} - \zeta \mathbf{E} = \mathbf{J}^p \end{aligned} \tag{1}$$

$$\begin{aligned} &\nabla \times \mathbf{E}^p - i\omega\mu_0 \mathbf{H}^p = 0 \\ &\nabla \times \mathbf{H}^p - \zeta_1 \mathbf{E}^p = \mathbf{J}^p \end{aligned} \tag{2}$$

$$\begin{aligned} &\nabla \times \mathbf{E}^s - i\omega\mu_0 \mathbf{H}^s = 0 \\ &\nabla \times \mathbf{H}^s - \zeta_1 \mathbf{E}^s = \mathbf{J}^q \end{aligned} \tag{3}$$

$$\mathbf{J}^q = (\zeta(\mathbf{r}) - \zeta_1)\mathbf{E} \tag{4}$$

By applying an integral equation (IE) approach, the secondary field can be obtained by equation (5), in which $\mathbf{G}^E$ and $\mathbf{G}^H$ denote Green's tensor. Finally, the total electromagnetic field can be obtained by equation (6)[4-5].

$$\begin{aligned} &\mathbf{E}^s(\omega,\mathbf{r}) = \int_v \mathbf{G}^E(\omega,\mathbf{r},\mathbf{r}') \cdot \mathbf{J}^q(\mathbf{r}')dv' \\ &\mathbf{H}^s(\omega,\mathbf{r}) = \int_v \mathbf{G}^H(\omega,\mathbf{r},\mathbf{r}') \cdot \mathbf{J}^q(\mathbf{r}')dv' \end{aligned} \tag{5}$$

$$\begin{aligned} &\mathbf{E}(\omega,\mathbf{r}) = \mathbf{E}^p(\omega,\mathbf{r}) + \int_v \mathbf{G}^E(\omega,\mathbf{r},\mathbf{r}') \cdot \mathbf{J}^q(\mathbf{r}')dv' \\ &\mathbf{H}(\omega,\mathbf{r}) = \mathbf{H}^p(\omega,\mathbf{r}) + \int_v \mathbf{G}^H(\omega,\mathbf{r},\mathbf{r}') \cdot \mathbf{J}^q(\mathbf{r}')dv' \end{aligned} \tag{6}$$

Inversion

The objective functional $\boldsymbol{\varphi}(\mathbf{m},\lambda)$ (equation (7)) can be written as a combination of data objective functional, $\boldsymbol{\varphi}_d(\mathbf{m})$, and model objective functional, $\boldsymbol{\varphi}_m(\mathbf{m})$,

$$\begin{aligned} \varphi(\mathbf{m},\lambda) &= \varphi_d(\mathbf{m}) + \lambda\varphi_m(\mathbf{m}) \\ &= \left\| \mathbf{d}^{obs} - \mathbf{d}^{cal} \right\|^2 + \lambda\varphi_m(\mathbf{m}) \end{aligned} \tag{7}$$

where $\mathbf{m}$ is the vector of model parameters ($\log(\sigma)$ in this case), $\mathbf{d}^{obs}$ is the vector of observed data ($\log(Z^{obs})$ in this case), $\mathbf{d}^{cal}$ is the vector of calculated data ($\log(Z^{cal})$ in this case), and λ is a hyper parameter for the regularization[6].

The quasi-Newton method is used to optimize the objective functional. This approach is a kind of Newton method with simplified calculation of the Hessian matrix by using BFGS update[7-8].

Synthetic test

Though our final goal is application to seafloor observation, we start building a code for the case of land observation. We assume a model of the Earth structure. It is composed of the first layer with heterogeneous conductivity and 1-D layers below. The first layer consists of many cubes with variable conductivity, and the value of each cube's conductivity is given by a random number. And the conductivity of each layer bellowing the first layer is homogeneous like in fig.1. The 25 observation sites are equably located all over the area of the surface fig.2. The model size is about 80*80*101(km^3); 16 periods range from 1s to 1000s.

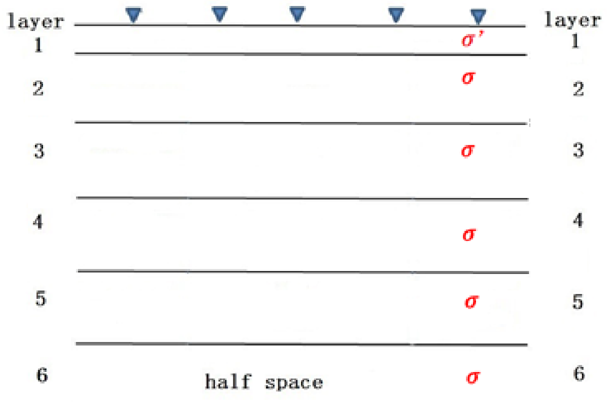

Fig.1 section view of the synthetic model

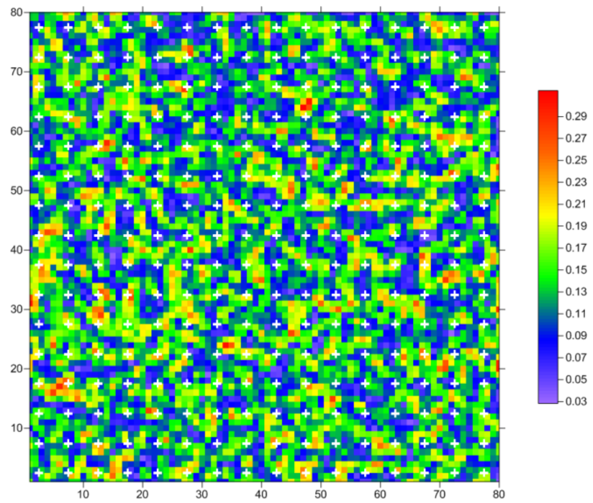

Fig.2 first layer vertical view and conductivity value

We tested the scheme by using synthetic data. In this case, a homogeneous half space is given below the first heterogeneous layer. In the testing case, the inversion was converged even without regularization. And reasonable agreement was obtained between inverted and synthetic responses (fig.3 -- fig.5). Figure 3(a) shows the synthetic model (black line) is a stair-like model; the initial inversion model (blue line) is a half space model; and the final inversion result (red line) is also one stair-like model. And the final inversion model is close to the synthetic model. Figure 3(b) shows the misfit changing trend, decreasing from value about 18 to value about 0.7.

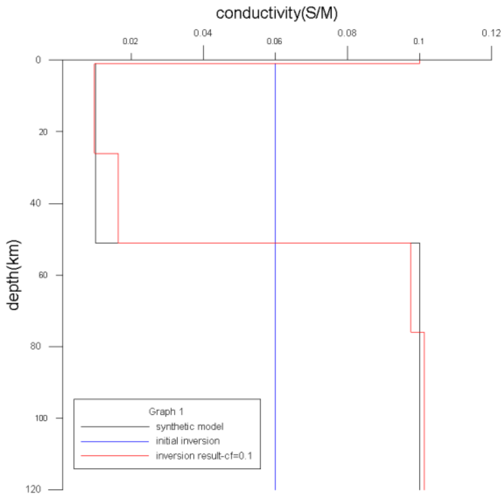

Fig.3 (a) synthetic, initial and final inversion model

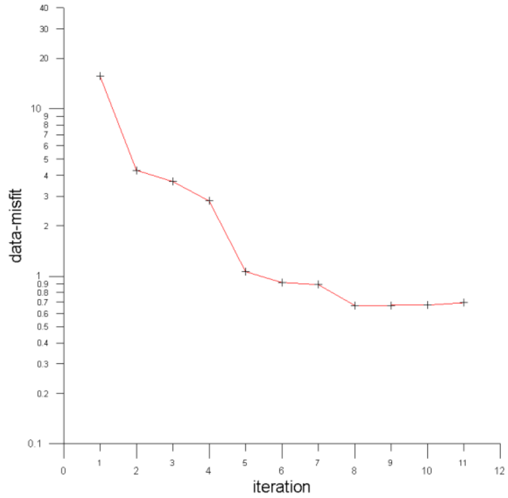

Fig.3 (b) misfit of the inversion process

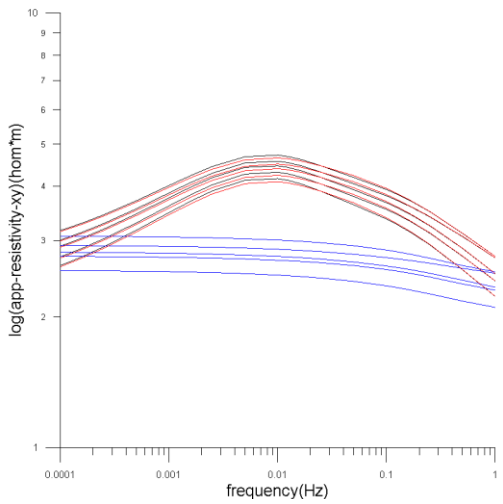

Fig.4 (a) apparent resistivity in xy direction

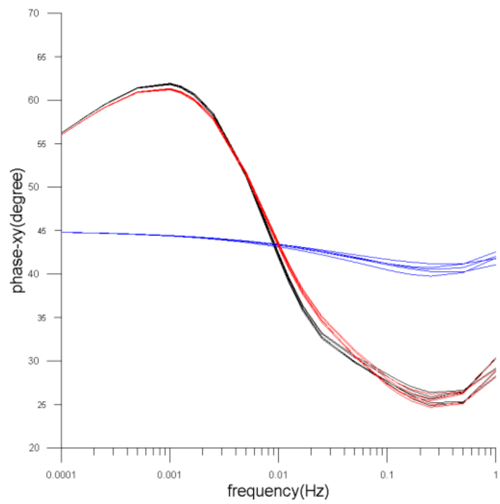

Fig.4 (b) phase in xy direction

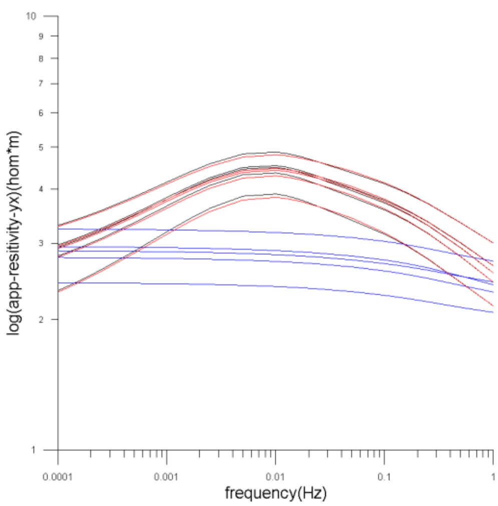

Fig.5 (a) apparent resistivity in yx direction

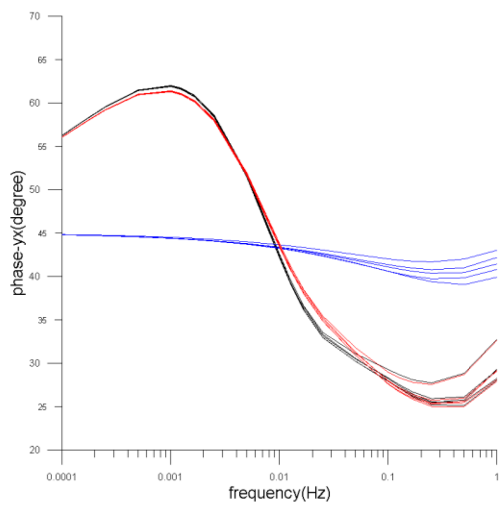

Fig.5 (b) phase in yx direction

Figure 4(a) shows the apparent resistivity of 5 sites from the synthetic model (black line), the initial inversion model (blue line) and the final inversion result (red line) in xy direction, respectively. Figure 4(b) shows the related phase of 5 sites from the synthetic model (black line), the initial inversion model (blue line) and the final inversion result (red line) in xy direction, respectively. Both these two figures show the sites' response and phase from the final inversion model is close to that from the synthetic model, respectively. These 5 sites are the diagonal sites from the top-left to the lower-right direction in the surface of first layer.

Figure 5 (a) shows the apparent resistivity of 5 sites from the synthetic model (black line), the initial inversion model (blue line) and the final inversion result (red line) in yx direction, respectively. Figure 5 (b) shows the related phase of 5 sites from the synthetic model (black line), the initial inversion model (blue line) and the final inversion result (red line) in yx direction, respectively. Both these two figures show the sites' response and phase from the final inversion model is close to that from the synthetic model, respectively. These 5 sites are the diagonal sites from the top-left to the lower-right direction in the surface of first layer.

Conclusion

The above case study shows the high reliability of this inversion method. However, the current model is built with flat first layer boundary and cannot reflect the real rough topography of the seafloor. So it is necessary to add the rough boundary in the synthetic testing in next step. And then it could be utilized to invert for 1-D background conductivity structures. And this 1-D conductivity structure model including first-layer-topography-effect could serve as starting model for 3-D inversion. In this way it is possible to obtain much more reliable 3-D inversion result.

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