Theoretical Magnetograms for S_q

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Summary

Electromagnetic induction by a hypothetical S_q in an Earth model consisting of a non-uniform surface sheet representing the oceans and an inner uniform spherical conductor is studied. It is found out that the electric currents induced in the surface sheet are larger than those for the case without the inner core. A resonance-like response of a non-uniform sheet is then suggested. Although expected magnetograms for a number of observatories are obtained, no good agreement between the calculated and the observed S_q is obtained because the present Earth model seems to lead to a considerable overestimate of the internal origin part.

1. Introduction

A theory of electromagnetic induction in a non-uniform spherical sheet, which simulates the land-sea distribution over the Earth's surface, placed on a non-conducting layer and underlain by a conducting sphere has been advanced by Rikitake (1968a) with an application to obtaining theoretical magnetograms for an idealized s.s.c. when the ocean effect is considered. Although no good agreement between theoretical and actual magnetograms has been achieved probably because of local irregularities, it is intended in this paper to extend a similar study to induction by an idealized S_q.

2. Electric currents induced in the surface sheet by an S_q

Most of the mathematics needed for the theory have already been described in the previous paper (Rikitake, 1968a). Let us start from equations (27) and (29) of the previous paper. Putting

$$p = i\alpha , \ (\alpha = 2\pi/T, \quad i = \sqrt{-1}),$$

where T is the period, equation (27) can be divided into two equations, the one for the real and the other for the imaginary parts. That is also the case for (29). We hence obtain four sets of simultaneous equations for real and imaginary parts of the induced current function coefficients, K_n^{mc} and K_n^{ms}. In actual calculation, spherical harmonic functions up to n = 3 and m = 3 are taken into account, so that it is required to solve a set of simultaneous equations with 30 unknowns.

It has been shown (Rikitake, 1967) that an inducing field of S_q, which is assumed to depend solely on local time and to consist of 24-and 12-hourly components only, can be expressed with four real and imaginary fields on the condition that proper choice of e_k^1 and ϵ_k^1 should be made. Accordingly, inductions by fields of which the potential are respectively expressed by $P_2^1 \cos \phi$, $P_2^1 \cos(\phi + \pi/2)$, $P_3^2 \cos 2\phi$ and $P_3^2 \cos(2\phi + \pi/2)$ are separately calculated. The induced part of S_q is then obtained by synthesizing these fields.

In Table 1 are given the main coefficients of the external potential of the yearly-averaged S_q during the IGY (Matsushita and Maeda, 1965).

Current function coefficients for the four inducing fields each having a unit amplitude are indicated in Tables 2a - 2d. The surface resistance distribution and the elements of the conducting core (q = 0.94 and $\sigma = 5 \times 10^{-12}$ e.m.u.) have already been described in the previous paper (1968a). In order order to see the influence of the conducting core, similar coefficients for the cases without the core (Rikitake, 1967) are also reproduced in Tables 3a - 3d. All these coefficients are determined by making use of semi-normalized Schmidt spherical functions.

3. Negative damping effect of the conducting core

That electric currents induced in a surface sheet are usually influenced by the conducting core in such a way that the current intensity in the sheet becomes small has often been pointed out notably by Rikitake (1961a, 1961b, 1966, 1968a). To the writers' surprise, however, it turns out by the present study that such a damping effect may not always prevail for a non-uniform sheet. On comparing Table 2 to Table 3, itis observed that the current function coefficients for $P_3^2 \cos 2\phi$ and $P_3^2 \cos (2\phi + \pi/2)$ for the case with the core are larger than those for the case without the core. We may therefore say that a negative damping effect, so to speak, of the conducting core is given rise to in some cases. Quite contrary to expectation, an augumentation of the ocean effect is derived by taking the core into account in the present model.

The reason why we have such a negative damping effect is not quite clear. Even if we assume perfect conductivity for the core of the same size, similar effect is observed. But, it we assume an inducing field of different type, i.e. $n = 1$ and $m = 0$, no such effect is derived. Rikitake's work (1968b) on electromagnetic induction in a non-uniform plane sheet in which electric resistance varies periodically in a direction shows that a resonance-like enhancement of the induced current is given rise to when the wavelength of the inducing field is about the same as that of the non-uniformity and the conductivity contrast in the sheet is very large. It is also proved that such a resonant phenomenon takes place when the period of variation is close to the time of free decay of electric currents in the sheet.

If we think of the Pacific Ocean, it would not be unreasonable to imagine that an inducing potential expressed by $P_3^2 \cos 2\phi$ would play an important role in the electromagnetic induction judging from the size and shape of the ocean. The time-constant of such a large ocean would amount to ten hours or thereabouts, so that necessary conditions for giving rise to an anomalous induction might be provided.

4. Distributions of the magnetic field

With the coefficients given in Table 2, the coefficients of magnetic potential arising from the interior of the Earth can readily be obtained by virtue of equation (34) in the previous paper (Rikitake, 1968a). The distribution of the internal potential obviously depends on universal time. It is noticed that the internal part becomes small for 8-12 h in universal time. In Fig. 1 is shown, for example, the equivalent current system for the internal S_q at 12 h in universal time.

The internal part of S_q thus estimated is definitely larger than that obtained by actual analyses (Rikitake et al., 1956; Price and Wilkins, 1963). It is therefore apparent that the internal S_q obtained on the basis of the present model is an over-estimate. Expected magnetograms for the three groups of observatories, for which s.s.c. magnetograms were calculated in the previous paper (Rikitake, 1968a), are nevertheless computed as shown in Figs. 2a - 2c. No good agreement between the observed and calculated S_q is naturally attained.

5. Discussion and concluding remarks

One of the most important points brought to light by the present study is certainly the anomalous enhancement of the induced current for a P_3^2 type inducing field. Close examination of a few analyses of the S_q (Chapman, 1919; Matsushita and Maeda, 1965) indicates that the ratio of the external coefficient to the internal one for n = 2 and m = 1 is always larger than that for n = 3 and m = 2, i.e. $(e_2^1/i_2^1)/(e_3^2/i_3^2)$ amounts to 1.4, 1.2 and 1.2 for the analyses for the equinoxes of 1902, 1905 and 1958. If this is true, the effect of induction is considerably larger for an inducing field of P_3^2 type than that of P_2^1 type. The writers suspect that such an effect might be attributed to the anomalous enhancement of the P_3^2 component as described in Section 3. The existing estimates of the electrical conductivity within the Earth having largely relied on the P_3^2 component of the S_q, it is feared that the conductivity and the size of the conducting core might have been over-estimated.

In Section 4 has been pointed out that the present model leads to an over-estimate of the internal part of S_q. One of the reasons of such an over-estimate would be the incorrect knowledge about the conducting core. But it is highly likely that the effect of the large conductivity contrast in the surface sheet, amounting to some $10^4 : 1$ at most, plays the most important role in resulting in the anomalous resonance-like response. The specific conductivity, especially that under the continents, has been taken rather arbitrarily, so that no good agreement between the observed and calculated S_q is obtained. Existing spherical harmonic analyses of S_q having been made on the assumption that the S_q depends solely on local time, no accurate determination of the main harmonic coefficients of the S_q has been possible because they depend on universal time as well as local time. In view of the possible pseudo-resonance of the surface conductors which might result in a fairly large amplitude of the S_q arising inside the Earth, it is highly desirable to analyse the S_q in such a way as to determine parts of the S_q which depend on universal time separately from those which depend on local time only.

References

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Table 1

Main coefficients in units of gamma of the external magnetic potential of the S_q

n	m	$e^m_{n,c}$	$e^m_{n,s}$
2	1	14.22	-0.01
3	2	-6.61	1.41

Table 2a

Coefficients of the induced current function for $P_2^1 \cos\phi$.

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	-0.06885	-0.01567	-	-
1	1	-0.01356	-0.00255	0.01065	0.00221
2	0	0.00780	0.00183	-	-
2	1	0.05148	0.01183	-0.00709	-0.00159
2	2	0.05588	0.01208	-0.03164	-0.00700
3	0	-0.00055	-0.00002	-	-
3	1	0.02745	0.00609	-0.00090	-0.00019
3	2	-0.03889	-0.00853	0.01723	0.00370
3	3	0.00078	0.00024	-0.00809	-0.00179

Table 2b

Coefficients of the induced current function for $P_2^1 \cos(\phi+\pi/2)$.

n	m	Pearl K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	-0.00599	-0.00130	-	-
1	1	-0.00116	-0.00008	0.00094	0.00025
2	0	0.00068	0.00017	-	-
2	1	0.00449	0.00104	-0.00064	-0.00027
2	2	0.00486	0.00100	-0.00280	-0.00083
3	0	-0.00005	0.00000	-	-
3	1	0.00239	0.00053	-0.00006	0.00007
3	2	-0.00339	-0.00074	0.00150	0.00031
3	3	0.00010	0.00016	-0.00069	-0.00006

Table 2c

Coefficients of the induced current function for $P_3^2 \cos 2\phi$

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	0.07815	0.01086	-	-
1	1	0.01545	0.00168	-0.01208	-0.00133
2	0	-0.00884	-0.00119	-	-
2	1	-0.05835	-0.00773	0.00804	0.00103
2	2	-0.06346	-0.00805	0.03590	0.00456
3	0	0.00064	-0.00003	-	-
3	1	-0.03114	-0.00397	0.00102	0.00016
3	2	0.04419	0.00605	-0.01958	-0.00248
3	3	-0.00087	-0.00007	0.00917	0.00109

Table 2d

Coefficients of the induced current function for $P_3^2 \cos(2\phi + \pi/2)$

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	0.01419	0.00170	-	-
1	1	0.00287	0.00070	-0.00227	-0.00075
2	0	-0.00161	-0.00026	-	-
2	1	-0.01058	-0.00108	0.00146	0.00019
2	2	-0.01155	-0.00148	0.00659	0.00123
3	0	0.00013	0.00007	-	-
3	1	-0.00567	-0.00071	0.00018	-0.00005
3	2	0.00805	0.00112	-0.00366	-0.00115
3	3	-0.00018	-0.00019	0.00164	0.00001

Table 3a

Coefficients of the induced current function for $P_2^1 \cos\phi$ for the case without core

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	-0.07527	-0.00694	-	-
1	1	-0.01489	0.00031	0.01167	0.00039
2	0	0.00852	0.00091	-	-
2	1	0.05625	0.00561	-0.00775	-0.00065
2	2	0.06117	0.00371	-0.03462	-0.00259
3	0	-0.00062	0.00030	-	-
3	1	0.03002	0.00235	-0.00098	-0.00006
3	2	-0.04254	-0.00303	0.01887	0.00111
3	3	0.00085	0.00026	-0.00885	-0.00066

Table 3b

Coefficients of the induced current function for $P_2^1 \cos(\phi + \pi/2)$ for the case without core

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	-0.00663	-0.00045	-	-
1	1	-0.00130	0.00044	0.00103	0.00022
2	0	0.00075	0.00011	-	-
2	1	0.00495	0.00054	-0.00068	-0.00047
2	2	0.00537	0.00020	-0.00306	-0.00091
3	0	-0.00005	0.00003	-	-
3	1	0.00264	0.00022	-0.00009	0.00025
3	2	-0.00374	-0.00026	0.00166	0.00007
3	3	0.00008	0.00046	-0.00077	0.00023

Table 3c

Coefficients of the induced current function for $P_3^2 \cos 2\phi$ for the case without core

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	0.06214	0.00360	-	-
1	1	0.01228	-0.00042	-0.00962	0.00036
2	0	-0.00704	-0.00022	-	-
2	1	-0.04642	-0.00181	0.00639	0.00015
2	2	-0.05047	-0.00114	0.02856	0.00059
3	0	0.00051	-0.00030	-	-
3	1	-0.02477	-0.00065	0.00081	0.00011
3	2	0.03511	0.00199	-0.01557	-0.00041
3	3	-0.00069	0.00010	0.00730	-0.00002

Table 3d

Coefficients of the induced current function for $P_3^2 \cos(2\phi + \pi/2)$ for the case without core

n	m	Real K_n^{mc}	Imag. K_n^{mc}	Real K_n^{ms}	Imag. K_n^{ms}
1	0	0.01128	-0.00006	-	-
1	1	0.00228	0.00087	-0.00181	-0.00118
2	0	-0.00128	-0.00014	-	-
2	1	-0.00842	0.00050	0.00116	0.00003
2	2	-0.00919	-0.00022	0.00524	0.00108
3	0	0.00010	0.00013	-	-
3	1	-0.00451	-0.00007	0.00014	-0.00017
3	2	0.00640	0.00039	-0.00029	-0.00179
3	3	-0.00014	-0.00042	0.00013	-0.00047

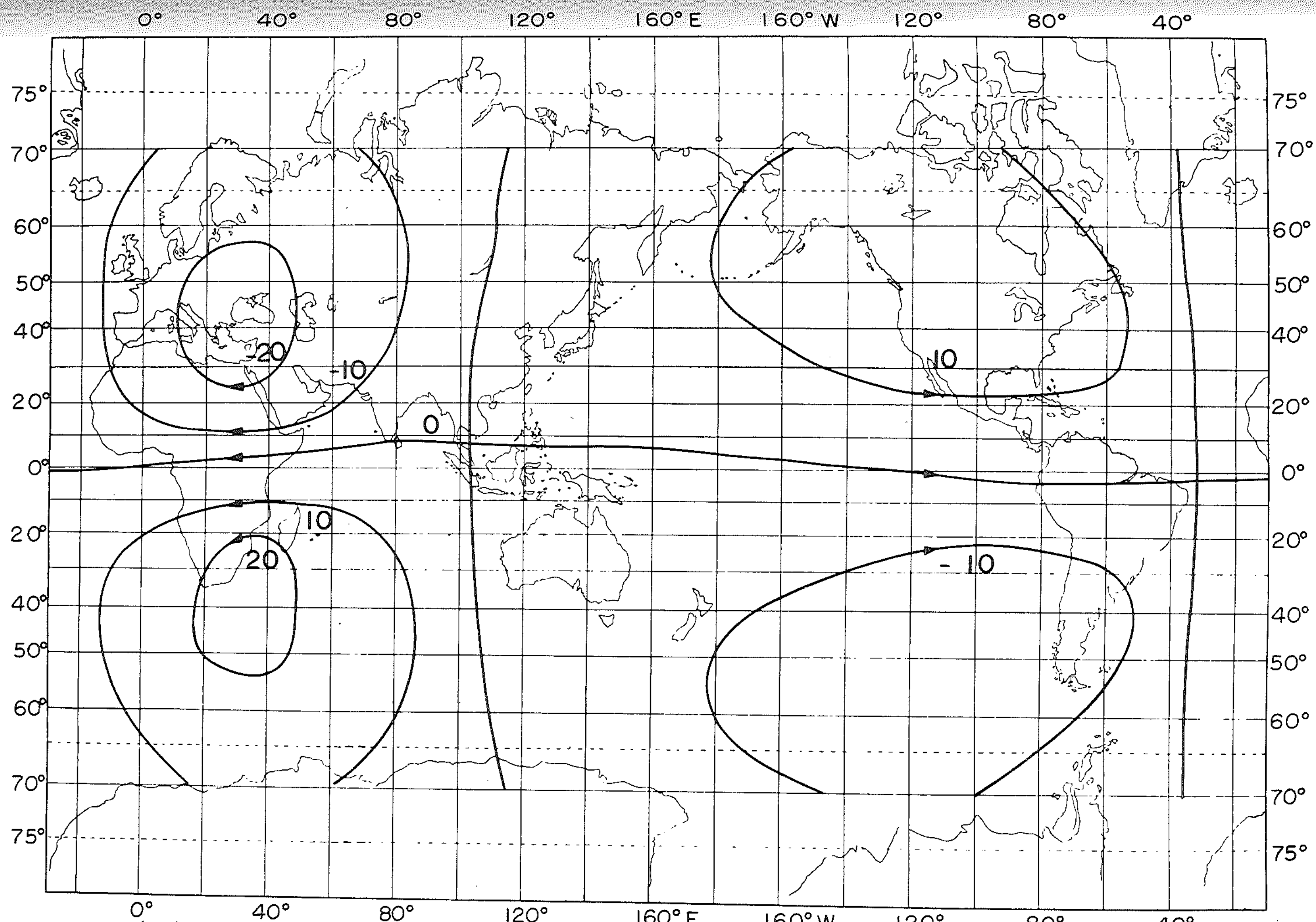

Fig. 1 Equivalent current system in units of 10^{-6} e. m. u. for the internal part of the S_q at 12 h UT.

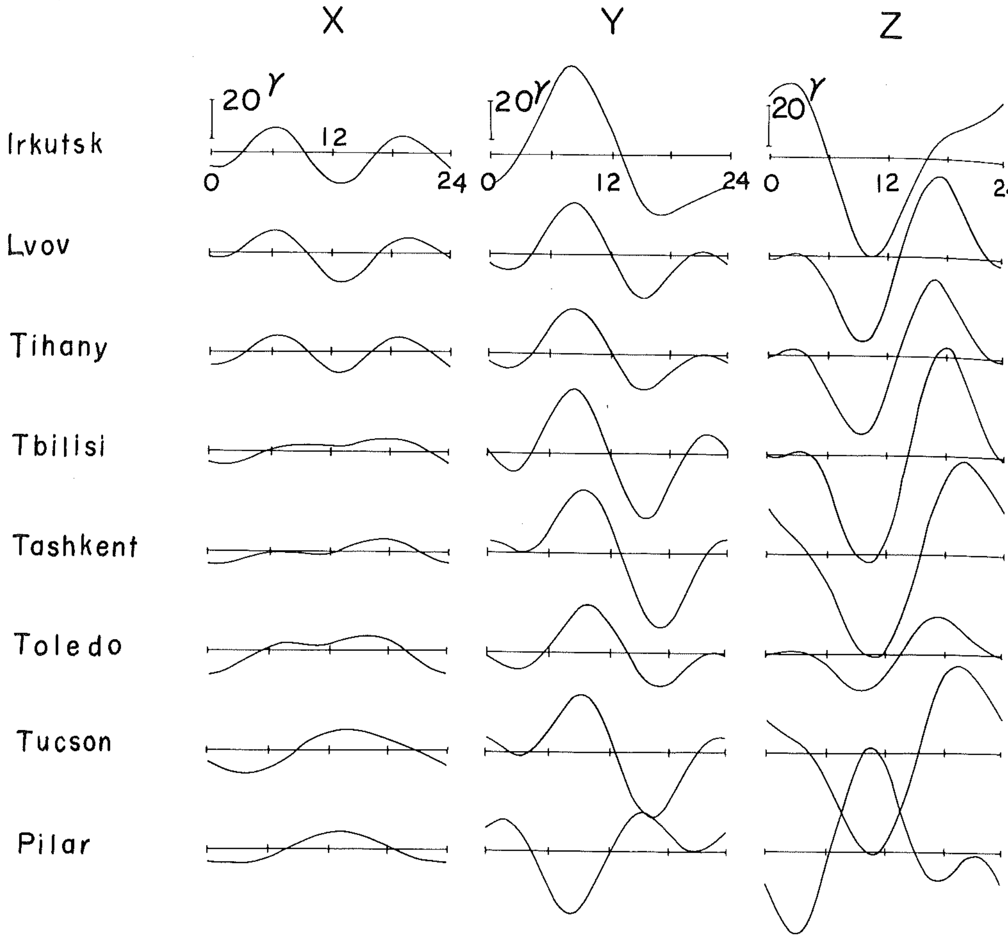

Fig. 2a Theoretical magnetograms for S_q at continental observatories.

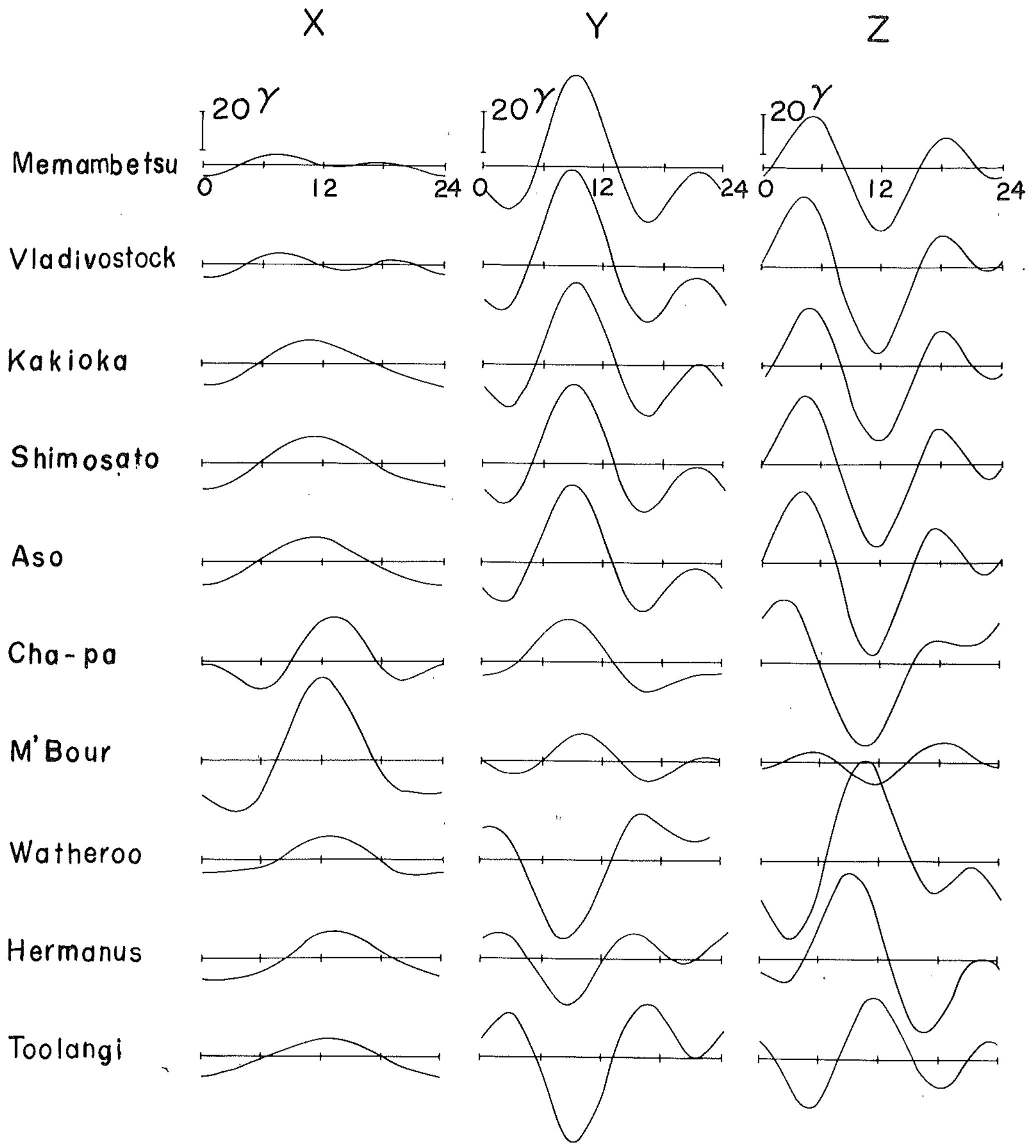

Fig. 2b Theoretical magnetograms at coastal observatories.

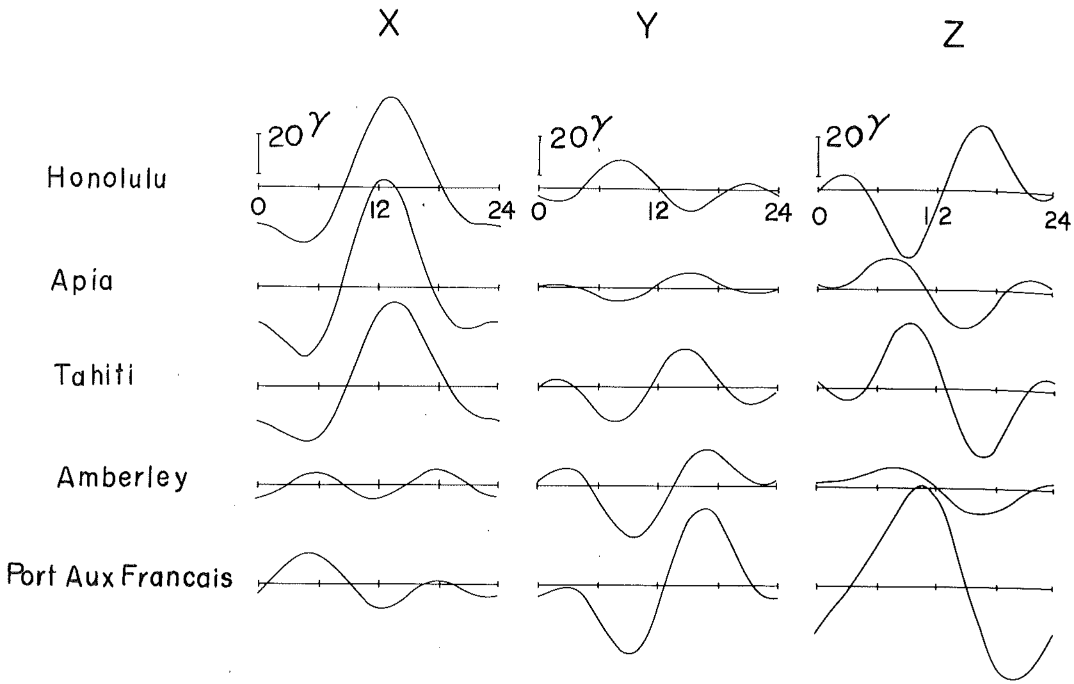

Fig. 2c Theoretical magnetograms at oceanic observatories.